Graphical Analysis of Motion

5-12 Distance-Time Graph of Motion at Constant Speed

The use of a graph is helpful in analyzing the motion of a body. To illustrate, let us apply it to an airplane moving at constant speed in a straight course at 100 meters per second. Table 5.1 lists the distance traveled by the airplane at the end of each of five 1-second intervals.

For each of the positions of the airplane, we plot a point on the graph in Fig. 5-9. The distance traveled by the plane is shown on the vertical axis and the travel time is shown on the horizontal axis. Point *A* represents the starting time of 0 seconds. Point *B* represents the 100-meter distance traveled by the plane at the end of 1.00 second. Its ordinate, or vertical distance from the horizontal axis, is 100 meters on the distance scale. Its abscissa, or horizontal distance from the vertical axis, is 1.00 second on the time scale. Point C represents the 200-meter distance traveled by the airplane at the end of 2.00 seconds. Its ordinate is 200 meters and its abscissa is 2.00 seconds. Points *D*, *E*, and *F* are obtained in a similar manner.

Now, connecting the six points, we notice that they all fall on a straight line. As we have seen earlier, a straight-line graph between two quantities shows that the quantities are proportional to each other. In this case, the distance-time graph shows that the distance traveled by the body is directly proportional to the travel time. That is, as the travel time is doubled, the distance is doubled; as the travel time is tripled, the distance is tripled, and so on.

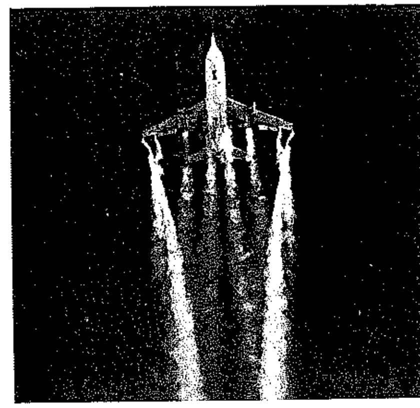

Fig. 5-8. Contrails can be used to determine the path of this airplane in space and time.

Table 5.1

TRAVEL TIME (s)	DISTANCE TRAVELED (m)
0.00	000
1.00	100
2.00	200
3.00	300
4.00	400
5.00	500

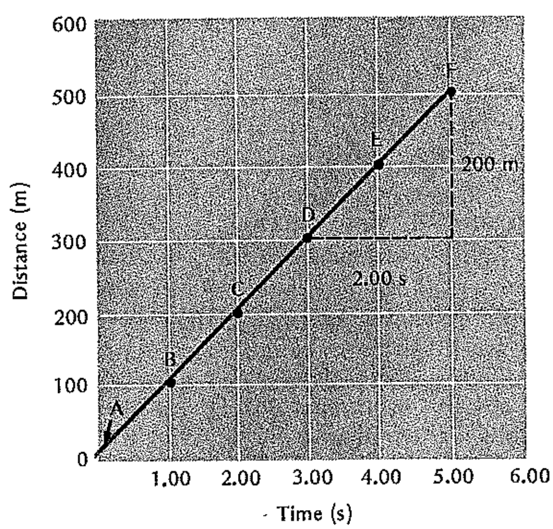

Fig. 5-9. A distance-time graph of motion at constant speed is a straight line.

5-13 Slope Represents the Speed

The slope or slant of the distance-time graph represents the speed, in this case, 100 meters per second. Let us show this.

In geometry, the slope of a line tells how much it is inclined to the horizontal axis. The slope is found by taking any two points on the line and dividing the difference between their ordinates by the difference between their abscissas.

For example, let us find the slope of the distance-time line from the points *F* and *D*. On the vertical scale, the ordinate of *F* represents a 500-meter distance and that of *D* represents a 300-meter distance. This gives a 200-meter difference in distance traveled. On the time scale, the abscissa of *F* represents a time 5.00 seconds after starting and the abscissa of *D* represents a time 3.00 seconds after starting. This gives a difference of 2.00 seconds in travel time. Dividing the 200-meter difference in the ordinates of *D* and *F* by the 2.00-second difference of their abscissas, we have 200 meters ÷ 2.00 seconds = 100 meters per second. This checks with the given speed of the airplane.

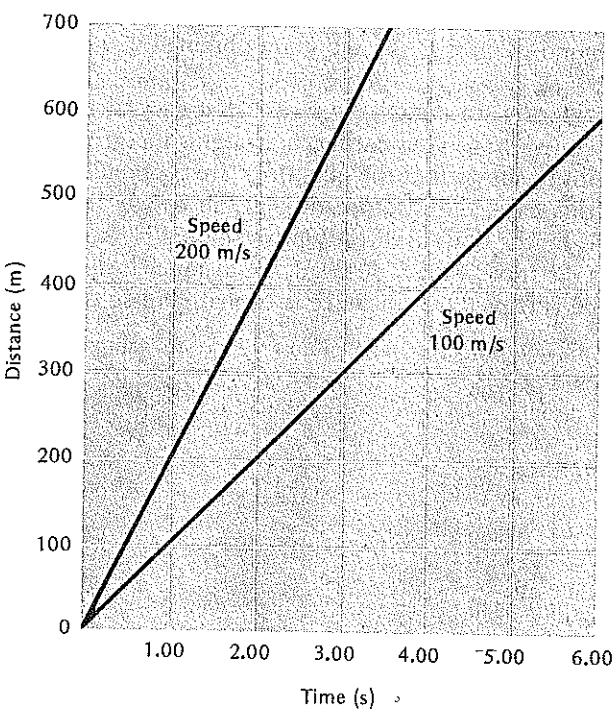

Fig. 5-10. The slope of the distance-time graph represents the speed.

The greater the slope of the distance-time graph, the greater is the speed it represents. In Fig. 5-10 are shown the distance-time graphs for two planes, one traveling at 100 meters per second and the other traveling at 200 meters per second. Note that the slope of the line representing the faster plane is steeper than that representing the slower plane.

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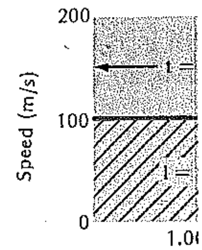

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5-16 Finding Spee

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5-14 Speed-Time Graph of Motion at Constant Speed

In Fig. 5-11, the constant speed of the plane flying at 100 meters per second is plotted against the time of travel. The speed is shown on the vertical axis and the time is shown on the horizontal axis. Since the speed is constant, all the points on the speed-time graph are the same distance above the horizontal axis and the line drawn through them is parallel to the horizontal axis.

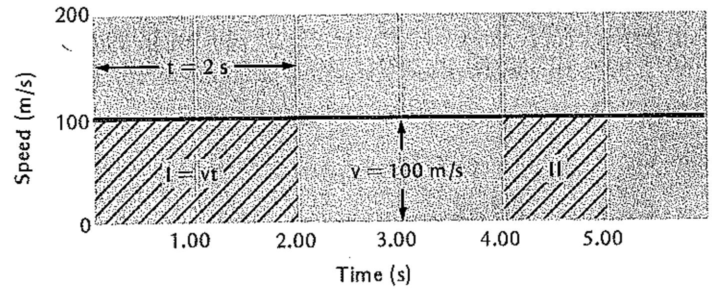

Fig. 5-11. A speed-time graph of motion at constant speed is a line parallel to the horizontal axis.

A particularly useful feature of this graph is the fact that *the area between the speed-time line and the horizontal axis represents the distance traveled by the body up to that time.* This is evident from rectangle I. Its vertical side is the speed v, and its horizontal side is the time of travel t. Its area is therefore $v \times t$ or vt, which is equal to the distance traveled by a body moving at constant speed v for a time t. The area of rectangle I gives us the distance traveled by the plane in 2.00 seconds. Since v is 100 meters per second and t is 2.00 seconds, the area is $100 \times 2.00 = 200$. This represents a distance of 200 meters. The area of rectangle II shows the distance covered in the time between the fourth and fifth seconds. This is $100 \times 1.00 = 100$ meters.

5-15 Distance-Time Graph of Uniformly Accelerated Motion

As an example of graphic analysis of uniformly accelerated motion, consider the motion of a body starting from rest and moving with an acceleration of 2.00 m/s^2. Table 5.2 lists the distances traveled by the body at the end of each of the first five seconds.

Table 5.2

TIME (s)	DISTANCE (m)
0.00	0.00
1.00	1.00
2.00	4.00
3.00	9.00
4.00	16.00
5.00	25.00

In Fig. 5-12, the distance is plotted as an ordinate and the time of fall as an abscissa for each second. Unlike the case of the body moving at constant speed, the distance-time graph is not a straight line but a curve. This curve, known as a *parabola* (puh·RAB·uh·luh) shows graphically the fact that the distance is directly proportional to the square of the time. It expresses the relationship $d = \frac{1}{2} at^2$.

5-16 Finding Speed from the Slope

The slope of the tangent to the distance-time graph at any point P is the instantaneous speed at that point. This can be seen from Fig.

5-12 where P_1 represents a position of the body a short time t_1 before it reaches P, and P_2 represents the position of the body a short time t_2 after it leaves P. The slope of the line P_1P_2 is the distance traveled by the body in going from P_1 to P_2 divided by the time, $t_2 - t_1$. It therefore represents the average speed of the body between P_1 and P_2. If P_1 and P_2 are taken closer and closer to P, the line P_1P_2 coincides with the tangent to the curve at P. Its slope then is equal to the instantaneous speed of the body at P.

Notice that the slope of the tangent at Q is steeper than that at P, which, in turn, is steeper than the slope of the tangent at N. This shows how the speed increases steadily as we go from N to P to Q.

Thus, an increasing slope in the distance-time graph indicates that the body is being positively accelerated. The numerical value of the slope of the tangent to the curve at any point may be determined exactly as was done in Section 5-13 by taking any two points on the tangent line and dividing the difference of their ordinates by the difference of their abscissas. Figure 5-12 shows how this is done for point P, at which the slope of the tangent turns out to be 12.00 meters divided by 2.00 seconds, or 6.00 meters per second. Using this procedure, the speeds at the end of each of the first 5 seconds have been determined and are shown in Table 5.3.

Table 5.3

TIME (s)	SPEED (m/s)
0.00	0.00
1.00	2.00
2.00	4.00
3.00	6.00
4.00	8.00
5.00	10.00

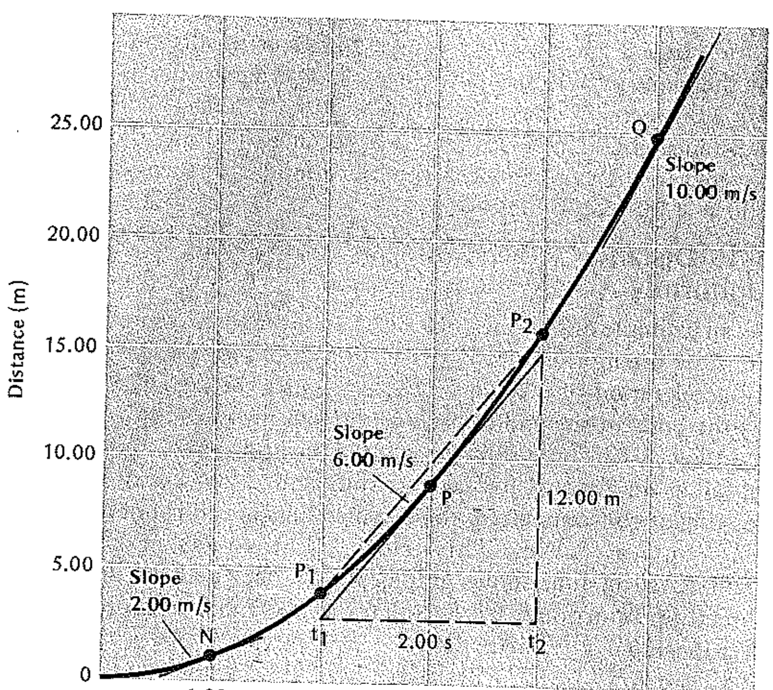

Fig. 5-12. Distance-time graph of uniformly accelerated motion.

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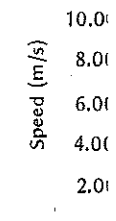

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5-18 Distance-Tim

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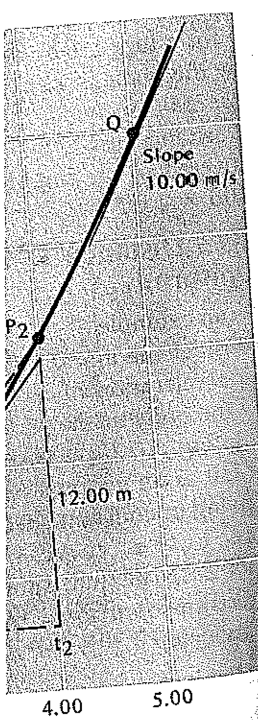


5-17 Speed-Time Graph of Uniformly Accelerated Motion

In Fig. 5-13, the speeds listed in Table 5.3 are plotted as ordinates against the time as abscissas. The speed-time graph is a straight line showing that the speed is proportional to the travel time. As in the case of motion at constant speed, the area under the speed-time line gives the distance covered by the body. This can be seen from the shaded triangle. Its horizontal leg is the time t. Its vertical leg is the speed at time t, which is equal to $v = at$. Its area is one-half the product of its legs, or $\frac{1}{2}\, at \times t = \frac{1}{2}\, at^2$. This is also the expression for the distance d which the body moves in time t.

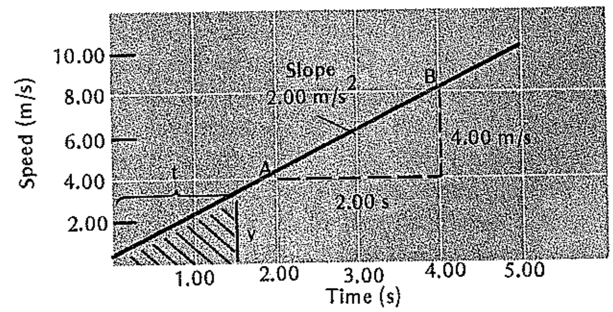


Fig. 5-13. A speed-time graph of uniformly accelerated motion is a straight line.

Just as the slope of the distance-time line gives the speed of the body at each point, so *the slope of the speed-time line gives the acceleration of the body at each point.* This can be seen by finding the slope from the points A and B in Fig. 5-13. The difference in the ordinates of the points A and B is 8.00 − 4.00 = 4.00 meters per second. The difference of the abscissas of A and B is 4.00 − 2.00 = 2.00 seconds. Hence, the slope is equal to 4.00 meters per second ÷ 2.00 seconds = 2.00 meters per second per second. This is the acceleration of this body.

5-18 Distance-Time Graph of Any Motion

Using the graphic method, we can now analyze any motion, whether uniformly or not uniformly accelerated. We usually have a record of the distance traveled by the moving body during a given time. We can plot this record as a distance-time graph. The slope of the tangent to this graph at any point will then tell us the instantaneous speed of the body at the time represented by that point.

To illustrate, the distance-time graph of a moving body is shown in Fig. 5-14. The straight-line section AB shows that the body moved at constant speed from $t = 0$ to $t = 2$ seconds. This speed is given by the slope of AB, which can be found from points A and B as follows.

The ordinate of B is 4 meters and that of A is 0 meters. The difference in ordinates is therefore 4 − 0 = 4 meters. The abscissa of B is 2 seconds and that of A is 0 seconds. The difference of abscissas is 2 − 0 = 2 seconds. The slope of AB is the difference

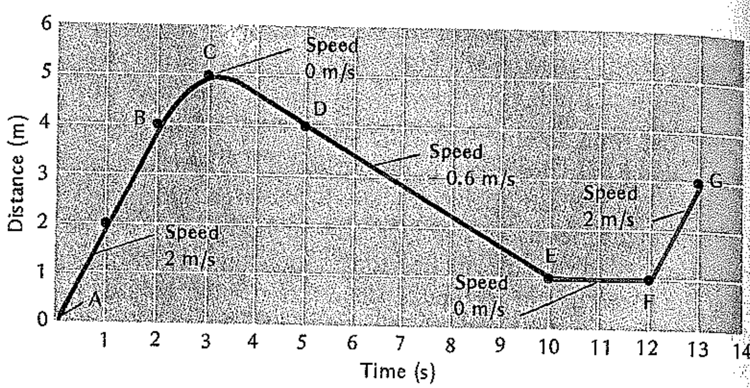


Fig. 5-14. Analysis of motion by means of a distance-time graph.

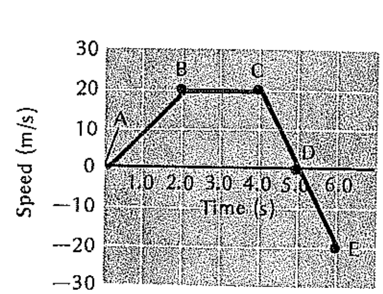
of the ordinates of A and B divided by the difference in their abscissas, or 4 meters ÷ 2 seconds = 2 meters per second. The speed of the body from A to B was 2 meters per second.

Between B and D the distance traveled increased to 5 meters at C, then decreased to 4 meters at D. This means that the body slowed up, stopped, and then reversed its direction of motion. The speed of the body at C is given by the slope of the tangent at C. Since this tangent is parallel to the time axis, it has zero slope. This shows that the instantaneous speed of the body at C is zero, or that the body was momentarily at rest at C.

From D to E the body again moved at constant speed, as shown by the slope of the straight-line segment DE. We obtain this slope from the points D and E as follows. The difference of the ordinates of E and D is 1 meter − 4 meters = −3 meters. The difference in the abscissas of E and D is 10 seconds − 5 seconds = 5 seconds. The slope of DE is therefore −3 meters divided by 5 seconds or −0.6 meter per second. Since the slope is negative, the speed it represents is opposite in direction to the body's original direction of motion. Thus, between D and E the body moved back toward its starting point at the speed of 0.6 meter per second.

At E, the body stopped moving and remained at rest between E and F. Finally, from F to G the body reversed direction again and moved at a constant speed of 2 meters per second.

The graph also gives information concerning the acceleration of the body. On each of the straight-line sections AB, DE, EF, and FG, the speed was constant or zero. The acceleration of the body while on each of these straight sections was therefore zero. Between B and D, the changing slope of the curve shows that the body underwent a negative acceleration that first decreased its speed to zero and then sped it up in the opposite direction.



5-19 Speed-Time Graph of Any Motion

In Fig. 5-15, the speed-time graph of a typical body is shown. It is seen that the slope of AB is 10 meters per second per second, the slope of BC is zero, and the slope of CD is −20 meters per second

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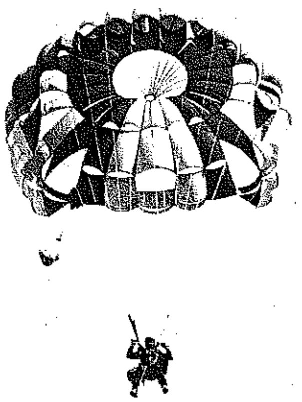

Fig. 5-19. Sport parachutists make use of air resistance to slow their parachutes down to a safe landing speed.

exposed to the air, like a feather or a sheet of paper, the effect of the air in slowing down its rate of fall is immediately seen. Parachutists take advantage of this slowing action of the air on the large surface of their parachutes to reduce their rate of fall to a safe speed.

When a falling object is fairly dense and has a comparatively small surface, like a stone or a solid block of iron, the slowing action of the air is small and is not readily noticed. Such an object falling through a small height will be accelerated by gravity very much like a freely falling body. However, when such a body falls from a great height, the air resistance upon it increases steadily, as its speed of fall increases.

After a time, the air resistance may become so large that it prevents any further increase in the speed of the body. The body then continues the remainder of its fall at constant speed. Thus, a body dropped from an airplane from a very great height will keep speeding up until it reaches a certain maximum velocity, called its *terminal velocity.* After that it will continue to fall at this maximum velocity without further change.

CHAPTER REVIEW

Summary

To describe the motion of a body, a **reference point** or **frame of reference** must first be chosen. The position of the body at any instant is then given by the body's displacement from the reference point. Its motion is given by its velocity with respect to the reference point.

A body may be in either uniform or accelerated motion. It is in **uniform motion** when its velocity is constant in both magnitude and direction. It is in **accelerated motion** when its velocity is changing in either direction or magnitude or both. The **acceleration** of a body at any instant is defined as the rate of change of its velocity. It is a vector quantity.

Motion in a straight line is particularly important because all more complicated motions can be considered to be composed of combinations of straight-line motions. A body in uniform motion moves in a straight line at constant speed. Such a body travels equal distances during equal intervals of time.

A body in **uniformly accelerated motion** in a straight line is one that has constant acceleration. Such a body either increases or decreases its speed at a constant rate.

For a body having an initial speed v_o and a constant acceleration a, the speed v at the end of a time interval t is given by:

$$\mathbf{v = v_o + at}$$

The distance d traveled by such a body is given by:

$$\mathbf{d = v_o t + \tfrac{1}{2} at^2}$$

These two relationships combine to give:

$$\mathbf{v^2 - v_o^2 = 2\,ad}$$

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A body falling freely
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Bodies do not fall freel
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Questions

Group 1

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Problems

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2. A car is moving at a
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For a body starting from rest, these relationships become:

$$v = at$$
$$d = \frac{1}{2} at^2$$
$$v^2 = 2\,ad$$

A **body falling freely** under the force of gravity undergoes uniformly accelerated motion. The constant **acceleration of gravity** (g) is equal to 9.80 meters per second per second, or 980 centimeters per second per second.

Bodies do not fall freely through air. They encounter frictional resistance that slows down their rate of fall and prevents their speeding up beyond a certain **terminal velocity.**

Questions

Group 1

1. In the distance-time graph of a body's motion, what does the slope at any point tell about the body's motion?
2. In the speed-time graph of a body's motion, what do each of the following tell about the body's motion: (*a*) the area under the speed-time line; (*b*) the slope of the speed-time line at any point?
3. The distance-time graph of a body's motion is a straight line making an acute angle with the time axis. What does this tell you about (*a*) the body's speed; (*b*) its acceleration?
4. The speed-time graph of a body is a straight line making an acute angle with the time axis. What does this tell you about (*a*) the body's speed; (*b*) its acceleration?
5. A sheet of paper and a pencil are pushed off a table at the same time and fall to the floor. (*a*) Why are the rates of fall different? (*b*) What is meant by the terminal velocity of a falling body? (*c*) Under what conditions would the downward acceleration of these two bodies be the same?
6. Explain Galileo's thought experiment showing that freely falling bodies of the same material should fall in exactly the same way.

Group 2

7. (*a*) For a certain time interval, the distance-time graph of a body's motion is parallel to the time axis. What does this tell about the body's motion during that time interval? (*b*) Why is it not possible for any part of the distance-time graph of a body to be a straight-line segment parallel to the distance axis? (*Hint:* What would such a straight-line segment indicate about the time taken to travel the distance it represents?)
8. (*a*) For a certain time interval, the speed-time graph of a body's motion is parallel to the time axis. What does this tell about the body's motion during that time interval? (*b*) Is it possible for any part of the speed-time graph to be parallel to the speed axis? Explain.

Problems

Group 1

1. The distance traveled by a car over a 10-s time interval increased as shown in Table 5.4. (*a*) Plot the distance-time graph of the motion. (*b*) What does it reveal about the motion of the car? (*c*) Determine the distance traveled by the car at the end of 5.5 s. (*d*) Determine the speed of the car.
2. A car is moving at a constant speed of 20 m/s. (*a*) Plot its speed-time graph for a 10.0-s interval.

Table 5.4

TIME (s)	DISTANCE (m)
0.0	0
2.0	35
4.0	70
6.0	105
8.0	140
10.0	175