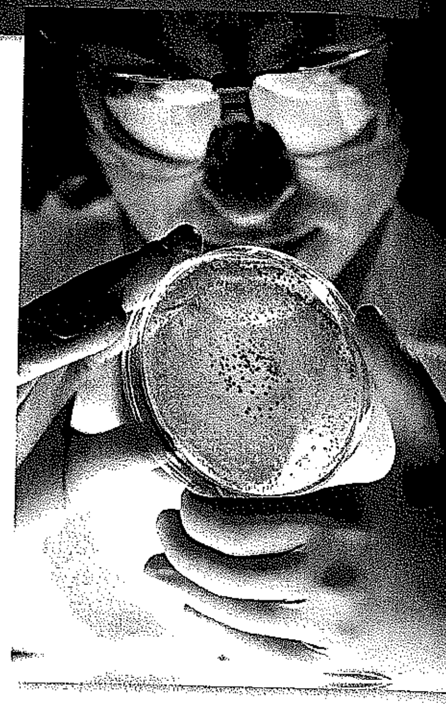
1-4 Powers and Exponents



Objective
Evaluate expressions containing exponents.

Vocabulary
power
base
exponent

Who uses this?
Biologists use exponents to model the growth patterns of living organisms.

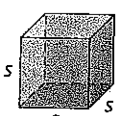
When bacteria divide, their number increases exponentially. This means that the number of bacteria is multiplied by the same factor each time the bacteria divide. Instead of writing repeated multiplication to express a product, you can use a power.

A **power** is an expression written with an *exponent* and a *base* or the value of such an expression. 3^2 is an example of a power.

The base, 3, is the number that is used as a factor. → 3^2 ← The exponent, 2, tells how many times the base, 3, is used as a factor.

When a number is raised to the second power, we usually say it is "squared." The area of a *square* is $s \cdot s = s^2$, where s is the side length.

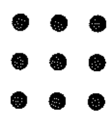
When a number is raised to the third power, we usually say it is "cubed." The volume of a *cube* is $s \cdot s \cdot s = s^3$, where s is the side length.



EXAMPLE 1 Writing Powers for Geometric Models

Write the power represented by each geometric model.

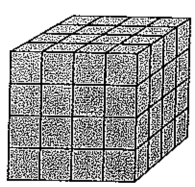
A



3^2

There are 3 rows of 3 dots. 3×3
The factor 3 is used 2 times.

B

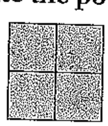


4^3

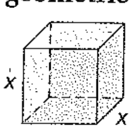
The figure is 4 cubes long, 4 cubes wide, and 4 cubes tall. $4 \times 4 \times 4$
The factor 4 is used 3 times.

CHECK IT OUT! **Write the power represented by each geometric model.**

1a.



1b.



There are no easy geometric models for numbers raised to exponents greater than 3, but you can still write them using repeated multiplication or a base and exponent.

Reading Exponents			
Words	**Multiplication**	**Power**	**Value**
3 to the first power	3	3^1	3
3 to the second power, or 3 squared	3 • 3	3^2	9
3 to the third power, or 3 cubed	3 • 3 • 3	3^3	27
3 to the fourth power	3 • 3 • 3 • 3	3^4	81
3 to the fifth power	3 • 3 • 3 • 3 • 3	3^5	243

EXAMPLE 2 Evaluating Powers

Simplify each expression.

A $(-2)^3$

$(-2)(-2)(-2)$ *Use −2 as a factor 3 times.*

-8

B -5^2

$-1 \cdot 5 \cdot 5$ *Think of a negative sign in front of a power as multiplying by −1. Find the product of −1 and two 5's.*

$-1 \cdot 25$

-25

C $\left(\frac{2}{3}\right)^2$

$\frac{2}{3} \cdot \frac{2}{3}$ *Use $\frac{2}{3}$ as a factor 2 times.*

$\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$

Caution!

In the expression -5^2, 5 is the base because the negative sign is not in parentheses.
In the expression $(-2)^3$, −2 is the base because of the parentheses.

CHECK IT OUT! **Simplify each expression.**

2a. $(-5)^3$ **2b.** -6^2 **2c.** $\left(\frac{3}{4}\right)^3$

EXAMPLE 3 Writing Powers

Write each number as a power of the given base.

A **8; base 2**

$2 \cdot 2 \cdot 2$ *The product of three 2's is 8.*

2^3

B **−125; base −5**

$(-5)(-5)(-5)$ *The product of three −5's is −125.*

$(-5)^3$

CHECK IT OUT! **Write each number as a power of the given base.**

3a. 64; base 8 **3b.** −27; base −3

EXAMPLE 4 *Problem-Solving Application*

PROBLEM SOLVING

A certain bacterium splits into 2 bacteria every hour. There is 1 bacterium on a slide. If each bacterium on the slide splits once per hour, how many bacteria will be on the slide after 6 hours?

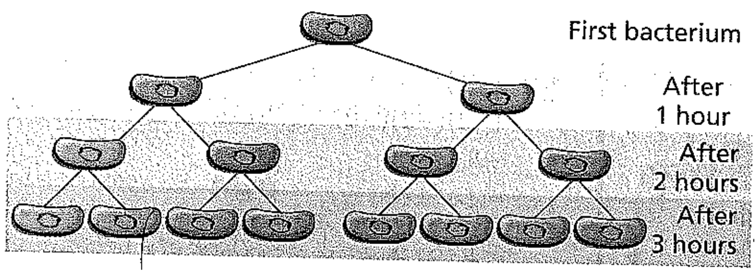
1 Understand the Problem

The **answer** will be the number of bacteria on the slide after 6 hours.
List the **important information:**

- There is 1 bacterium on a slide that divides into 2 bacteria.
- Each bacterium then divides into 2 more bacteria.

2 Make a Plan

Draw a diagram to show the number of bacteria after each hour.



3 Solve

Notice that after each hour, the number of bacteria is a power of 2.

After 1 hour: $1 \cdot 2 = 2$ or 2^1 bacteria on the slide
After 2 hours: $2 \cdot 2 = 4$ or 2^2 bacteria on the slide
After 3 hours: $4 \cdot 2 = 8$ or 2^3 bacteria on the slide

So, after the 6th hour, there will be 2^6 bacteria.

$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$ *Multiply six 2's.*

After 6 hours, there will be 64 bacteria on the slide.

4 Look Back

The numbers become too large for a diagram quickly, but a diagram helps you recognize a pattern. Then you can write the numbers as powers of 2.

CHECK IT OUT! **4. What if...?** How many bacteria will be on the slide after 8 hours?

THINK AND DISCUSS

1. Express 8^3 in words two ways.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example and tell whether the expression is positive or negative.

	Even Exponent	Odd Exponent
Positive Base		
Negative Base		

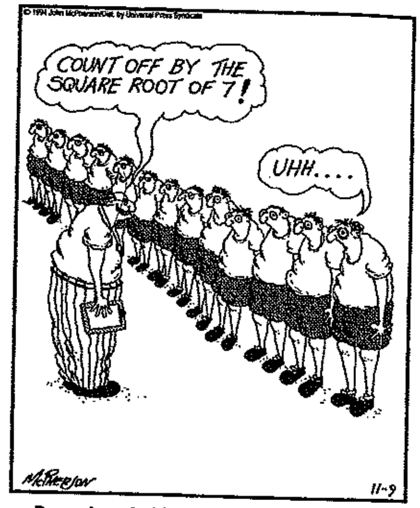
1-5 Square Roots and Real Numbers

Objectives
Evaluate expressions containing square roots.

Classify numbers within the real number system.

Vocabulary
square root
perfect square
real numbers
natural numbers
whole numbers
integers
rational numbers
terminating decimal
repeating decimal
irrational numbers

Why learn this?
Square roots are used to find the side length of a square when you know the area of the square, like when covering a square plot with flower seeds. (See Example 2.)



Deep down inside, Coach Knott had always wanted to be a math teacher.

A number that is multiplied by itself to form a product is called a **square root** of that product. The operations of squaring and finding a square root are inverse operations.

The radical symbol, $\sqrt{}$, is used to represent square roots. Positive real numbers have two square roots.

$$4 \cdot 4 = 4^2 = 16 \longrightarrow \sqrt{16} = 4 \longleftarrow$$ Positive square root of 16

$$(-4)(-4) = (-4)^2 = 16 \rightarrow -\sqrt{16} = -4 \leftarrow$$ Negative square root of 16

The nonnegative square root is represented by $\sqrt{}$. The negative square root is represented by $-\sqrt{}$.

A **perfect square** is a number whose positive square root is a *whole number*. Some examples of perfect squares are shown in the table.

0	1	4	9	16	25	36	49	64	81	100
0^2	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2

Reading Math

The expression $\sqrt{-36}$ does not represent a real number because there is no real number that can be multiplied by itself to form a product of -36.

EXAMPLE 1 Finding Square Roots of Perfect Squares

Find each square root.

A $\sqrt{49}$

$7^2 = 49$ — *Think: What number squared equals 49?*

$\sqrt{49} = 7$ — *Positive square root → positive 7*

B $-\sqrt{36}$

$6^2 = 36$ — *Think: What is the opposite of the square root of 36?*

$-\sqrt{36} = -6$ — *Negative square root → negative 6*

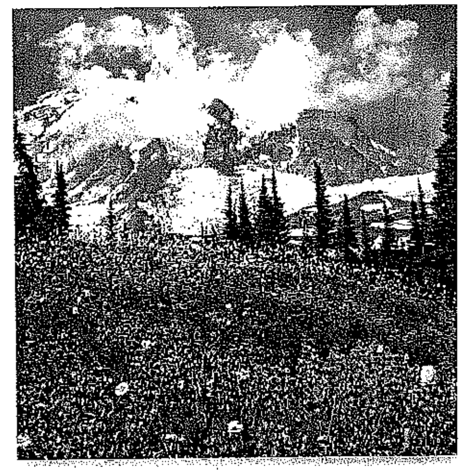
CHECK IT OUT! **Find each square root.**

1a. $\sqrt{4}$ **1b.** $-\sqrt{25}$

The square roots of many numbers, like $\sqrt{15}$, are not whole numbers. A calculator can approximate the value of $\sqrt{15}$ as 3.872983346... Without a calculator, you can use square roots of perfect squares to help estimate the square roots of other numbers.

EXAMPLE 2 *Problem-Solving Application*

PROBLEM SOLVING



Nancy wants to plant wildflowers in a square-shaped plot. She has enough wildflower seeds to cover 19 ft^2. Estimate to the nearest tenth the side length of a square plot with an area of 19 ft^2.

1 Understand the Problem

The **answer** will be the side length of the square garden.

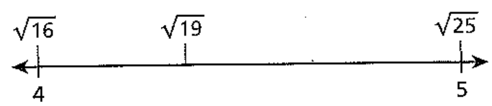
List the **important information:**

- The garden has an area of 19 feet.

2 Make a Plan

The side length of the square is $\sqrt{19}$ because $\sqrt{19} \cdot \sqrt{19} = 19$. 19 is not a perfect square, so $\sqrt{19}$ is not a whole number. Estimate $\sqrt{19}$ to the nearest tenth.

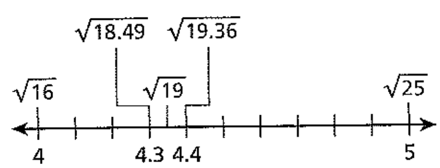
Find the two whole numbers that $\sqrt{19}$ is between. 19 is between the perfect squares 16 and 25, so $\sqrt{19}$ is between $\sqrt{16}$ and $\sqrt{25}$, or between 4 and 5. 19 is closer to 16 than to 25, so $\sqrt{19}$ is closer to 4 than to 5.



You can use a guess-and-check method to estimate $\sqrt{19}$.

3 Solve

Guess 4.3:	$4.3^2 = 18.49$	too low	*$\sqrt{19}$ is greater than 4.3.*
Guess 4.4:	$4.4^2 = 19.36$	too high	*$\sqrt{19}$ is less than 4.4.*



Because 19 is closer to 19.36 than to 18.49, $\sqrt{19}$ is closer to 4.4 than to 4.3.

$\sqrt{19} \approx 4.4$

Writing Math

The symbol $\approx$ means approximately equal to.

4 Look Back

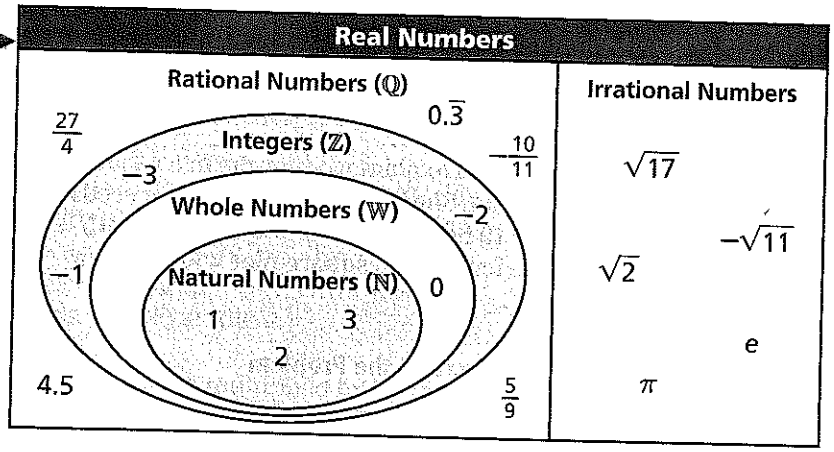
A square garden with a side length of 4.4 ft would have an area of 19.36 ft^2. 19.36 is close to 19, so 4.4 ft is a reasonable estimate.

CHECK IT OUT!

2. **What if...?** Nancy decides to buy more wildflower seeds and now has enough to cover 38 ft^2. What is the side length of a square garden with an area of 38 ft^2?

All numbers that can be represented on the number line are called **real numbers** and can be classified according to their characteristics.

Know it! Note



- **Natural numbers** are the counting numbers: 1, 2, 3, ...
- **Whole numbers** are the natural numbers and zero: 0, 1, 2, 3, ...
- **Integers** are whole numbers and their opposites: −3, −2, −1, 0, 1, 2, 3, ...
- **Rational numbers** can be expressed in the form $\frac{a}{b}$, where a and b are both integers and $b \neq 0$: $\frac{1}{2}, \frac{7}{1}, \frac{9}{10}$
- **Terminating decimals** are rational numbers in decimal form that have a finite number of digits: 1.5, 2.75, 4.0
- **Repeating decimals** are rational numbers in decimal form that have a block of one or more digits that repeats continuously: $1.\overline{3}, 0.\overline{6}, 2.\overline{14}, 6.2\overline{7}$
- **Irrational numbers** cannot be expressed in the form $\frac{a}{b}$. They include square roots of whole numbers that are not perfect squares and nonterminating decimals that do not repeat: $\sqrt{2}, \sqrt{11}, \pi$

EXAMPLE 3 Classifying Real Numbers

Write all classifications that apply to each real number.

A $\frac{8}{9}$

$8 \div 9 = 0.888... = 0.\overline{8}$ *$\frac{8}{9}$ can be written as a repeating decimal.*

rational number, repeating decimal

B 18

$18 = \frac{18}{1} = 18.0$ *18 can be written as a fraction and a decimal.*

rational number, terminating decimal, integer, whole number, natural number

C $\sqrt{20}$

$\sqrt{20} = 4.472135...$ *The digits of $\sqrt{20}$ continue with no pattern.*

irrational number

Reading Math

Note the symbols for the sets of numbers.

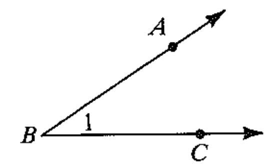
ℝ: real numbers
ℚ: rational numbers
ℤ: integers
𝕎: whole numbers
ℕ: natural numbers

CHECK IT OUT! **Write all classifications that apply to each real number.**

3a. $7\frac{4}{9}$ **3b.** −12 **3c.** $\sqrt{10}$

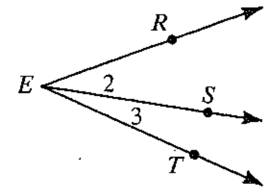
1-4 *Angles*

An **angle** (∠) is the figure formed by two rays that have the same endpoint. The two rays are called the **sides** of the angle, and their common endpoint is the **vertex** of the angle.



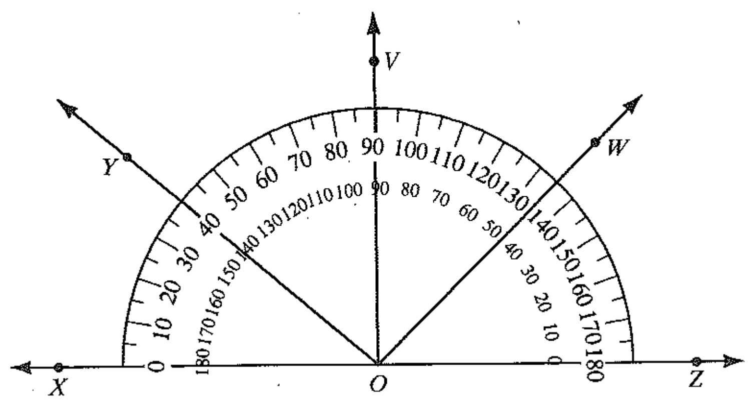
The sides of the angle shown are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The vertex is point B. The angle can be called $\angle B$, $\angle ABC$, $\angle CBA$, or $\angle 1$. If three letters are used to name an angle, the middle letter must name the vertex.

When you talk about this $\angle B$, everyone knows what angle you mean. But if you tried to talk about $\angle E$ in the diagram at the right, people wouldn't know which angle you meant. There are three angles with vertex E. To name any particular one of them you need to use either three letters or a number.



$\angle 2$ could also be called $\angle RES$ or $\angle SER$.
$\angle 3$ could also be called $\angle SET$ or $\angle TES$.
$\angle RET$ could also be called $\angle TER$.

You can use a protractor like the one shown below to find the *measure in degrees* of an angle. Although angles are sometimes measured in other units, this book will always use degree measure. Using the outer (red) scale of the protractor, you can see that $\angle XOY$ is a 40° angle. You can indicate that the (degree) measure of $\angle XOY$ is 40 by writing $m\angle XOY = 40$.



Using the inner scale of the protractor, you find that:

$m\angle YOZ = 140 \quad m\angle WOZ = 45 \quad m\angle YOW = 140 - 45 = 95$

Angles are classified according to their measures.

Acute angle:	Measure between 0 and 90	**Examples:** $\angle XOY$ and $\angle VOW$
Right angle:	Measure 90	**Examples:** $\angle XOV$ and $\angle VOZ$
Obtuse angle:	Measure between 90 and 180	**Examples:** $\angle XOW$ and $\angle YOW$
Straight angle:	Measure 180	**Example:** $\angle XOZ$

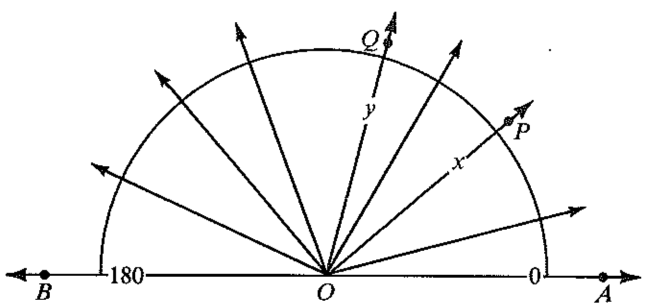
The two angle postulates below are very much like the Ruler Postulate and the Segment Addition Postulate on page 12.

Postulate 3 *Protractor Postulate*

On $\overleftrightarrow{AB}$ in a given plane, choose any point O between A and B. Consider $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and all the rays that can be drawn from O on one side of $\overleftrightarrow{AB}$. These rays can be paired with the real numbers from 0 to 180 in such a way that:

a. $\overrightarrow{OA}$ is paired with 0, and $\overrightarrow{OB}$ with 180.

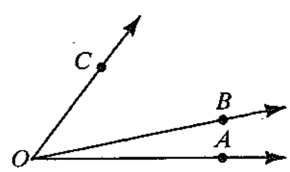
b. If $\overrightarrow{OP}$ is paired with x, and $\overrightarrow{OQ}$ with y, then $m\angle POQ = |x - y|$.



Postulate 4 *Angle Addition Postulate*

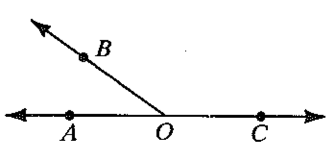
If point B lies in the interior of $\angle AOC$, then

$$m\angle AOB + m\angle BOC = m\angle AOC.$$



If $\angle AOC$ is a straight angle and B is any point not on $\overleftrightarrow{AC}$, then

$$m\angle AOB + m\angle BOC = 180.$$



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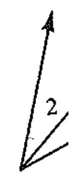

The **bisector**
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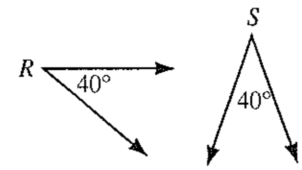
a right angle

Congruent angles are angles that have equal measures. Since $\angle R$ and $\angle S$ both have measure 40, you can write

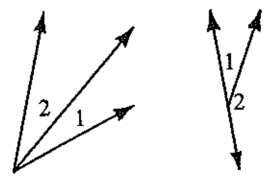
$$m\angle R = m\angle S \text{ or } \angle R \cong \angle S.$$

The definition of congruent angles tells us that these two statements are equivalent. We will use them interchangeably.

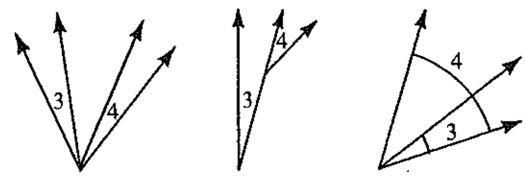


Adjacent angles (adj. $\angle$s) are two angles in a plane that have a common vertex and a common side but no common interior points.

$\angle 1$ and $\angle 2$ are adjacent angles.



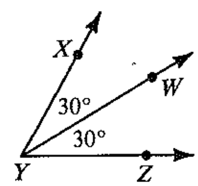
$\angle 3$ and $\angle 4$ are not adjacent angles.



The **bisector of an angle** is the ray that divides the angle into two congruent adjacent angles. In the diagram,

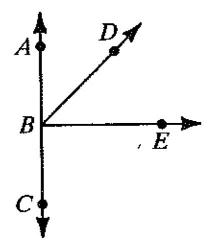
$$m\angle XYW = m\angle WYZ,$$
$$\angle XYW \cong \angle WYZ,$$

and $\overrightarrow{YW}$ bisects $\angle XYZ$.



There are certain things that you can conclude from a diagram and others that you can't. The following are things you can conclude from the diagram shown below.

All points shown are coplanar.
$\overleftrightarrow{AB}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at B.
A, B, and C are collinear.
B is between A and C.
$\angle ABC$ is a straight angle.
D is in the interior of $\angle ABE$.
$\angle ABD$ and $\angle DBE$ are adjacent angles.



The diagram above does *not* tell you that $\overline{AB} \cong \overline{BC}$, that $\angle ABD \cong \angle DBE$, or that $\angle CBE$ is a right angle. These three new pieces of information can be indicated in a diagram by using marks as shown at the right. Note that a small square is used to indicate a right angle (rt. $\angle$).