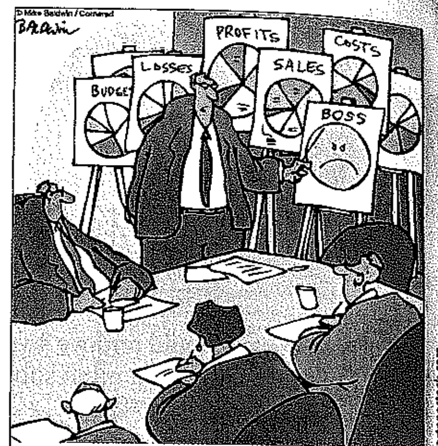
7-6 Adding and Subtracting Polynomials

Objective
Add and subtract polynomials.

Who uses this?

Business owners can add and subtract polynomials that model profit. (See Example 4.)



"This one pretty much sums it up."

Just as you can perform operations on numbers, you can perform operations on polynomials. To add or subtract polynomials, combine like terms.

Remember!

Like terms are constants or terms with the same variable(s) raised to the same power(s). To review combining like terms, see Lesson 1-7.

EXAMPLE 1 Adding and Subtracting Monomials

Add or subtract.

A $15m^3 + 6m^2 + 2m^3$

$15m^3 + 6m^2 + 2m^3$ — *Identify like terms.*

$15m^3 + 2m^3 + 6m^2$ — *Rearrange terms so that like terms are together.*

$17m^3 + 6m^2$ — *Combine like terms.*

B $3x^2 + 5 - 7x^2 + 12$

$3x^2 + 5 - 7x^2 + 12$ — *Identify like terms.*

$3x^2 - 7x^2 + 5 + 12$ — *Rearrange terms so that like terms are together.*

$-4x^2 + 17$ — *Combine like terms.*

C $0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$

$0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$ — *Identify like terms.*

$0.9y^5 - 0.4y^5 + y^5 + 0.5x^5$ — *Rearrange terms so that like terms are together.*

$1.5y^5 + 0.5x^5$ — *Combine like terms.*

D $2x^2y - x^2y - x^2y$

$2x^2y - x^2y - x^2y$ — *All terms are like terms.*

0 — *Combine.*

CHECK IT OUT! Add or subtract.

1a. $2s^2 + 3s^2 + s$

1b. $4z^4 - 8 + 16z^4 + 2$

1c. $2x^8 + 7y^8 - x^8 - y^8$

1d. $9b^3c^2 + 5b^3c^2 - 13b^3c^2$

Polynomials can be added in either vertical or horizontal form.

In vertical form, align the like terms and add:

$$\begin{array}{r} 5x^2 + 4x + 1 \\ +\ 2x^2 + 5x + 2 \\ \hline 7x^2 + 9x + 3 \end{array}$$

In horizontal form, use the Associative and Commutative Properties to regroup and combine like terms:

$$(5x^2 + 4x + 1) + (2x^2 + 5x + 2)$$
$$= (5x^2 + 2x^2) + (4x + 5x) + (1 + 2)$$
$$= 7x^2 + 9x + 3$$

EXAMPLE 2 Adding Polynomials

Add.

A $(2x^2 - x) + (x^2 + 3x - 1)$

$(2x^2 - x) + (x^2 + 3x - 1)$ *Identify like terms.*

$(2x^2 + x^2) + (-x + 3x) + (-1)$ *Group like terms together.*

$3x^2 + 2x - 1$ *Combine like terms.*

B $(-2ab + b) + (2ab + a)$

$(-2ab + b) + (2ab + a)$ *Identify like terms.*

$(-2ab + 2ab) + b + a$ *Group like terms together.*

$0 + b + a$ *Combine like terms.*

$b + a$ *Simplify.*

Writing Math

When you use the Associative and Commutative Properties to rearrange the terms, the sign in front of each term must stay with that term.

C $(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$

$(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$ *Identify like terms.*

$(4b^5 + 8b) + (-4b^5 + 7b)$ *Combine like terms in the second polynomial.*

$$\begin{array}{r} 4b^5 + 8b \\ \underline{+ -4b^5 + 7b} \end{array}$$ *Use the vertical method.*

$0 \quad + 15b$ *Combine like terms.*

$15b$ *Simplify.*

D $(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$

$(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$ *Identify like terms.*

$$\begin{array}{r} 20.2y^2 + 6y + 5 \\ \underline{+ 1.7y^2 + 0y - 8} \end{array}$$ *Use the vertical method.* *Write 0y as a placeholder in the second polynomial.*

$21.9y^2 + 6y - 3$ *Combine like terms.*

CHECK IT OUT! 2. Add $(5a^3 + 3a^2 - 6a + 12a^2) + (7a^3 - 10a)$.

To subtract polynomials, remember that subtracting is the same as adding the opposite. To find the opposite of a polynomial, you must write the opposite of *each* term in the polynomial:

$$-(2x^3 - 3x + 7) = -2x^3 + 3x - 7$$

EXAMPLE 3 Subtracting Polynomials

Subtract.

A $(2x^2 + 6) - (4x^2)$

$(2x^2 + 6) + (-4x^2)$ *Rewrite subtraction as addition of the opposite.*

$(2x^2 + 6) + (-4x^2)$ *Identify like terms.*

$(2x^2 - 4x^2) + 6$ *Group like terms together.*

$-2x^2 + 6$ *Combine like terms.*

B $(a^4 - 2a) - (3a^4 - 3a + 1)$

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Rewrite subtraction as addition of the opposite.*

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Identify like terms.*

$(a^4 - 3a^4) + (-2a + 3a) - 1$ *Group like terms together.*

$-2a^4 + a - 1$ *Combine like terms.*

Subtract.

C $(3x^2 - 2x + 8) - (x^2 - 4)$

$(3x^2 - 2x + 8) + (-x^2 + 4)$	*Rewrite subtraction as addition of the opposite.*
$(3x^2 - 2x + 8) + (-x^2 + 4)$	*Identify like terms.*
$3x^2 - 2x + 8$	*Use the vertical method.*
$+ -x^2 + 0x + 4$	*Write 0x as a placeholder.*
$2x^2 - 2x + 12$	*Combine like terms.*

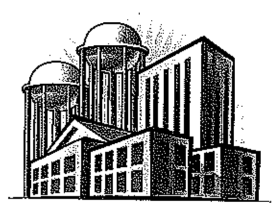
D $(11z^3 - 2z) - (z^3 - 5)$

$(11z^3 - 2z) + (-z^3 + 5)$	*Rewrite subtraction as addition of the opposite.*
$(11z^3 - 2z) + (-z^3 + 5)$	*Identify like terms.*
$11z^3 - 2z + 0$	*Use the vertical method.*
$+ -z^3 + 0z + 5$	*Write 0 and 0z as placeholders.*
$10z^3 - 2z + 5$	*Combine like terms.*

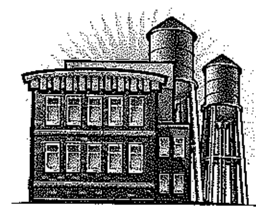
CHECK IT OUT! 3. Subtract $(2x^2 - 3x^2 + 1) - (x^2 + x + 1)$.

EXAMPLE 4 *Business Application*

The profits of two different manufacturing plants can be modeled as shown, where x is the number of units produced at each plant.



Eastern: $-0.03x^2 + 25x - 1500$



Southern: $-0.02x^2 + 21x - 1700$

Write a polynomial that represents the difference of the profits at the eastern plant and the profits at the southern plant.

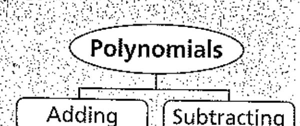
$(-0.03x^2 + 25x - 1500)$	*Eastern plant profits*
$-(-0.02x^2 + 21x - 1700)$	*Southern plant profits*
$(-0.03x^2 + 25x - 1500)$	
$+(+0.02x^2 - 21x + 1700)$	*Write subtraction as addition of the opposite.*
$-0.01x^2 + 4x + 200$	*Combine like terms.*

CHECK IT OUT! 4. Use the information above to write a polynomial that represents the total profits from both plants.

THINK AND DISCUSS

1. Identify the like terms in the following list: $-12x^2, -4.7y, \frac{1}{5}x^2y, y, 3xy^2, -9x^2, 5x^2y, -12x$
2. Describe how to find the opposite of $9t^2 - 5t + 8$.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example that shows how to perform the given operation.

Know it! Note



7-7 Multiplying Polynomials

Objective
Multiply polynomials.

Why learn this?
You can multiply polynomials to write expressions for areas, such as the area of a dulcimer. (See Example 5.)

To multiply monomials and polynomials, you will use some of the properties of exponents that you learned earlier in this chapter.

EXAMPLE 1 Multiplying Monomials

Multiply.

A $(5x^2)(4x^3)$

$(5x^2)(4x^3)$

$(5 \cdot 4)(x^2 \cdot x^3)$ — *Group factors with like bases together.*

$20x^5$ — *Multiply.*

B $(-3x^3y^2)(4xy^5)$

$(-3x^3y^2)(4xy^5)$

$(-3 \cdot 4)(x^3 \cdot x)(y^2 \cdot y^5)$ — *Group factors with like bases together.*

$-12x^4y^7$ — *Multiply.*

C $\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2} \cdot 6\right)(a^3 \cdot a^2)(b \cdot b^2)(c^2)$ — *Group factors with like bases together.*

$3a^5b^3c^2$ — *Multiply.*

Remember!
When multiplying powers with the same base, keep the base and add the exponents.
$x^2 \cdot x^3 = x^{2+3} = x^5$

CHECK IT OUT! **Multiply.**

1a. $(3x^3)(6x^2)$ **1b.** $(2r^2t)(5t^3)$ **1c.** $\left(\frac{1}{3}x^2y\right)(12x^3z^2)(y^4z^5)$

To multiply a polynomial by a monomial, use the Distributive Property.

EXAMPLE 2 Multiplying a Polynomial by a Monomial

Multiply.

A $5(2x^2 + x + 4)$

$5(2x^2 + x + 4)$

$(5)2x^2 + (5)x + (5)4$ — *Distribute 5.*

$10x^2 + 5x + 20$ — *Multiply.*

Multiply.

B $2x^2y(3x - y)$

$(2x^2y)(3x - y)$

$(2x^2y)3x + (2x^2y)(-y)$	*Distribute $2x^2y$.*
$(2 \cdot 3)(x^2 \cdot x)y + 2(-1)(x^2)(y \cdot y)$	*Group like bases together.*
$6x^3y - 2x^2y^2$	*Multiply.*

C $4a(a^2b + 2b^2)$

$4a(a^2b + 2b^2)$

$(4a)a^2b + (4a)2b^2$	*Distribute 4a.*
$(4)(a \cdot a^2)(b) + (4 \cdot 2)(a)(b^2)$	*Group like bases together.*
$4a^3b + 8ab^2$	*Multiply.*

CHECK IT OUT! **Multiply.**

2a. $2(4x^2 + x + 3)$ **2b.** $3ab(5a^2 + b)$ **2c.** $5r^2s^2(r - 3s)$

To multiply a binomial by a binomial, you can apply the Distributive Property more than once:

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$	*Distribute x and 3.*
$= x(x + 2) + 3(x + 2)$	
$= x(x) + x(2) + 3(x) + 3(2)$	*Distribute x and 3 again.*
$= x^2 + 2x + 3x + 6$	*Multiply.*
$= x^2 + 5x + 6$	*Combine like terms.*

Another method for multiplying binomials is called the FOIL method.

1. Multiply the **F**irst terms. F: $(x+3)(x+2) \rightarrow x \cdot x = x^2$
2. Multiply the **O**uter terms. O: $(x+3)(x+2) \rightarrow x \cdot 2 = 2x$
3. Multiply the **I**nner terms. I: $(x+3)(x+2) \rightarrow 3 \cdot x = 3x$
4. Multiply the **L**ast terms. L: $(x+3)(x+2) \rightarrow 3 \cdot 2 = 6$

$$(x+3)(x+2) = \underset{F}{x^2} + \underset{O}{2x} + \underset{I}{3x} + \underset{L}{6} = x^2 + 5x + 6$$

EXAMPLE 3 Multiplying Binomials

Multiply.

A $(x+2)(x-5)$

$(x+2)(x-5)$	
$x(x-5)+2(x-5)$	*Distribute x and 2.*
$x(x)+x(-5)+2(x)+2(-5)$	*Distribute x and 2 again.*
$x^2-5x+2x-10$	*Multiply.*
$x^2-3x-10$	*Combine like terms.*

B $(x+5)^2$

$(x+5)(x+5)$	*Write as a product of two binomials.*
$(x\cdot x)+(x\cdot 5)+(5\cdot x)+(5\cdot 5)$	*Use the FOIL method.*
$x^2+5x+5x+25$	*Multiply.*
$x^2+10x+25$	*Combine like terms.*

C $(3a^2-b)(a^2-2b)$

$3a^2(a^2)+3a^2(-2b)-b(a^2)-b(-2b)$	*Use the FOIL method.*
$3a^4-6a^2b-a^2b+2b^2$	*Multiply.*
$3a^4-7a^2b+2b^2$	*Combine like terms.*

Helpful Hint

In the expression $(x+5)^2$, the base is $(x+5)$.

$(x+5)^2 =$
$(x+5)(x+5)$

CHECK IT OUT! **3a.** $(a+3)(a-4)$ **3b.** $(x-3)^2$ **3c.** $(2a-b^2)(a+4b^2)$

To multiply polynomials with more than two terms, you can use the Distributive Property several times. Multiply $(5x+3)$ by $(2x^2+10x-6)$:

$$(5x+3)(2x^2+10x-6) = 5x(2x^2+10x-6)+3(2x^2+10x-6)$$
$$= 5x(2x^2+10x-6)+3(2x^2+10x-6)$$
$$= 5x(2x^2)+5x(10x)+5x(-6)+3(2x^2)+3(10x)+3(-6)$$
$$= 10x^3+50x^2-30x+6x^2+30x-18$$
$$= 10x^3+56x^2-18$$

You can also use a rectangle model to multiply polynomials with more than two terms. This is similar to finding the area of a rectangle with length $(2x^2+10x-6)$ and width $(5x+3)$:

	$2x^2$	$+10x$	-6
$5x$	$10x^3$	$50x^2$	$-30x$
$+3$	$6x^2$	$30x$	-18

Write the product of the monomials in each row and column.

To find the product, add all of the terms inside the rectangle by combining like terms and simplifying if necessary.

$$10x^3+6x^2+50x^2+30x-30x-18$$
$$10x^3+56x^2-18$$

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Another method that can be used to multiply polynomials with more than two terms is the vertical method. This is similar to methods used to multiply whole numbers.

$$\begin{array}{rrrrl} & 2x^2 & +10x & -6 & \\ \times & & 5x & +3 & \\ \hline & 6x^2 & +30x & -18 & \textit{Multiply each term in the top polynomial by 3.} \\ +\,10x^3 & +50x^2 & -30x & & \textit{Multiply each term in the top polynomial by 5x, and align like terms.} \\ \hline 10x^3 & +56x^2 & +0x & -18 & \textit{Combine like terms by adding vertically.} \\ 10x^3 & +56x^2 & & -18 & \textit{Simplify.} \end{array}$$

EXAMPLE 4 Multiplying Polynomials

Helpful Hint

A polynomial with *m* terms multiplied by a polynomial with *n* terms has a product that, before simplifying, has *mn* terms. In Example 4A, there are 2 • 3, or 6, terms before simplifying.

Multiply.

A $(x+2)(x^2-5x+4)$

$(x+2)(x^2-5x+4)$

$x(x^2-5x+4)+2(x^2-5x+4)$ — *Distribute x and 2.*

$x(x^2)+x(-5x)+x(4)+2(x^2)+2(-5x)+2(4)$ — *Distribute x and 2 again.*

$x^3+2x^2-5x^2-10x+4x+8$ — *Simplify.*

x^3-3x^2-6x+8 — *Combine like terms.*

B $(3x-4)(-2x^3+5x-6)$

$(3x-4)(-2x^3+5x-6)$

$$\begin{array}{rrrrrl} & -2x^3 & +0x^2 & +5x & -6 & \textit{Add } 0x^2 \textit{ as a placeholder.} \\ \times & & & 3x & -4 & \\ \hline & 8x^3 & +0x^2 & -20x & +24 & \textit{Multiply each term in the top polynomial by } -4. \\ +\,-6x^4 & +0x^3 & +15x^2 & -18x & & \textit{Multiply each term in the top polynomial by 3x, and align like terms.} \\ \hline -6x^4 & +8x^3 & +15x^2 & -38x & +24 & \textit{Combine like terms by adding vertically.} \end{array}$$

C $(x-2)^3$

$[(x-2)(x-2)](x-2)$ — *Write as the product of three binomials.*

$[x \cdot x+x(-2)-2 \cdot x-2(-2)](x-2)$ — *Use the FOIL method on the first two factors.*

$(x^2-2x-2x+4)(x-2)$ — *Multiply.*

$(x^2-4x+4)(x-2)$ — *Combine like terms.*

$(x-2)(x^2-4x+4)$ — *Use the Commutative Property of Multiplication.*

$x(x^2-4x+4)+(-2)(x^2-4x+4)$ — *Distribute x and −2.*

$x(x^2)+x(-4x)+x(4)+(-2)(x^2)$
$\quad +(-2)(-4x)+(-2)(4)$ — *Distribute x and −2 again.*

$x^3-4x^2+4x-2x^2+8x-8$ — *Simplify.*

$x^3-6x^2+12x-8$ — *Combine like terms.*

Multiply.

D $(2x+3)(x^2-6x+5)$

	x^2	$-6x$	$+5$
$2x$	$2x^3$	$-12x^2$	$10x$
$+3$	$3x^2$	$-18x$	15

Write the product of the monomials in each row and column.

$2x^3+3x^2-12x^2-18x+10x+15$ *Add all terms inside the rectangle.*

$2x^3-9x^2-8x+15$ *Combine like terms.*

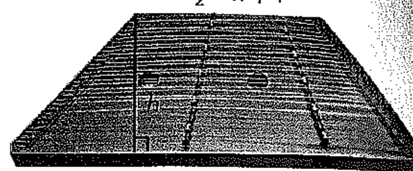
CHECK IT OUT! **Multiply.**

4a. $(x+3)(x^2-4x+6)$

4b. $(3x+2)(x^2-2x+5)$

EXAMPLE 5 *Music Application*

$b_2 = h + 1$



$b_1 = 2h - 1$

A dulcimer is a musical instrument that is sometimes shaped like a trapezoid.

A **Write a polynomial that represents the area of the dulcimer shown.**

$A=\frac{1}{2}h(b_1+b_2)$ *Write the formula for area of a trapezoid.*

$=\frac{1}{2}h[(2h-1)+(h+1)]$ *Substitute 2h − 1 for b_1 and h + 1 for b_2.*

$=\frac{1}{2}h(3h)$ *Combine like terms.*

$=\frac{3}{2}h^2$ *Simplify.*

The area is represented by $\frac{3}{2}h^2$.

B **Find the area of the dulcimer when the height is 22 inches.**

$A=\frac{3}{2}h^2$ *Use the polynomial from part a.*

$=\frac{3}{2}(22)^2$ *Substitute 22 for h.*

$=\frac{3}{2}(484)=726$

The area is 726 square inches.

CHECK IT OUT! **5.** The length of a rectangle is 4 meters shorter than its width.

a. Write a polynomial that represents the area of the rectangle.

b. Find the area of the rectangle when the width is 6 meters.

THINK AND DISCUSS

1. Compare the vertical method for multiplying polynomials with the vertical method for multiplying whole numbers.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, multiply two polynomials using the given method.